GPS bias*

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1 Problem description

Ten students of geoinformatics want to test GPS-based distance measurements. They (consecutively) record the GPS-coordinates of (the outer track of) the 100m starting line in an athletics stadium close by, then (consecutively) walk along the outer track till the finishing line, and again record the GPS-coordinates. Each of them repeats this procedure 50 times, Figure 1 depicts the first 10 measurements per person. For each of the 500 pairs they calculate the distance in meters. Given the sample size of $n = 500$ they expect the mean distance to be pretty close to 100m - all the bigger the surprise when the mean distance turns out to be roughly 102m. What went wrong - just bad luck?

We first model the situation mathematically and the answer the question with the help of simulations.

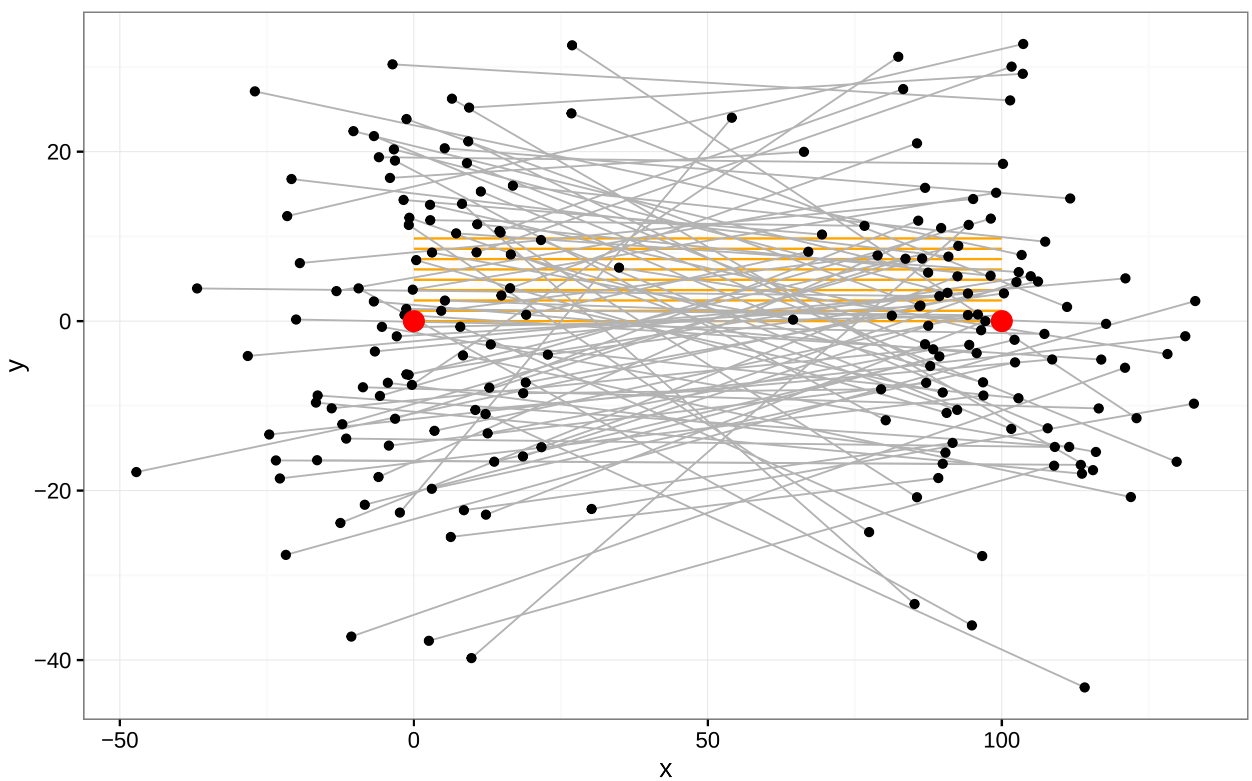

Figure 1: First 100 measurements

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2 Stochastic model for the measurements

Without loss of generality we assume that the starting point S and the end point Z have the following coordinates $S = (0, 0), Z = (100, 0)$. The points S', Z' will denote the measured coordinates; $F = (X_1, Y_1)$ denotes the measurement error in S, $G = (X_2, Y_2)$ the measurement error in Z. In other words:

$$S' = S + (X_1, Y_1) = (X_1, Y_1) \quad (1)$$
$$Z' = Z + (X_2, Y_2) = (100 + X_2, Y_2) \quad (2)$$

The measured distance is given by

$$\|S' - Z'\|_2 = \sqrt{(100 + X_2 - X_1)^2 + (Y_2 - Y_1)^2}. \quad (3)$$

3 Simulation results for normally distributed errors

3.1 The case $\sigma = 15$

To simplify matters we now assume $\sigma^2 = 15$ and simulate $R = 10.000$ samples $S'_1, S'_2, \ldots, S'_R$ of S' and $Z'_1, Z'_2, \ldots, Z'_R$ of Z'. Of each pair S'_i, Z'_i we then calculate the distance $d_i = \|S'_i - Z'_i\|_2$ and set $bias = \bar{d}_n - 100$.

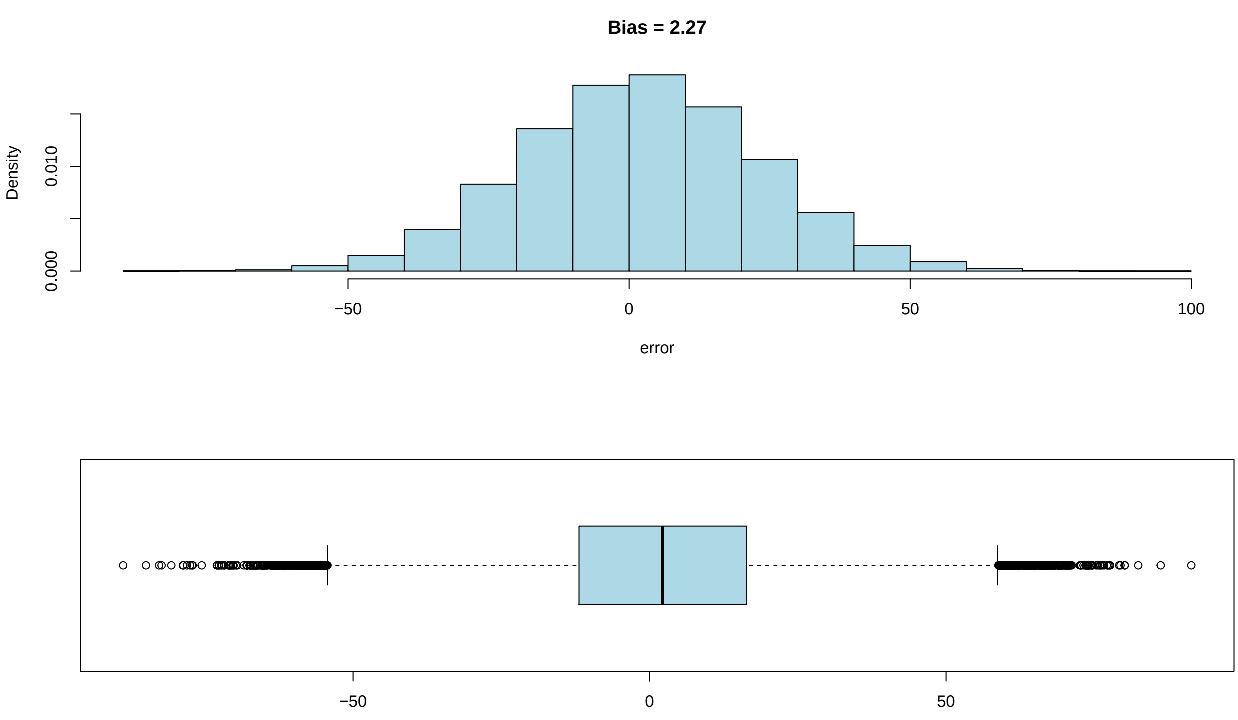


Figure 2: Histogram and boxplot of the observed errors for the case σ= 15

3.2 The case $\sigma^2 = 25$

Proceeding as before for the case $\sigma = 25$ we get the following results depicted in Figure 3.

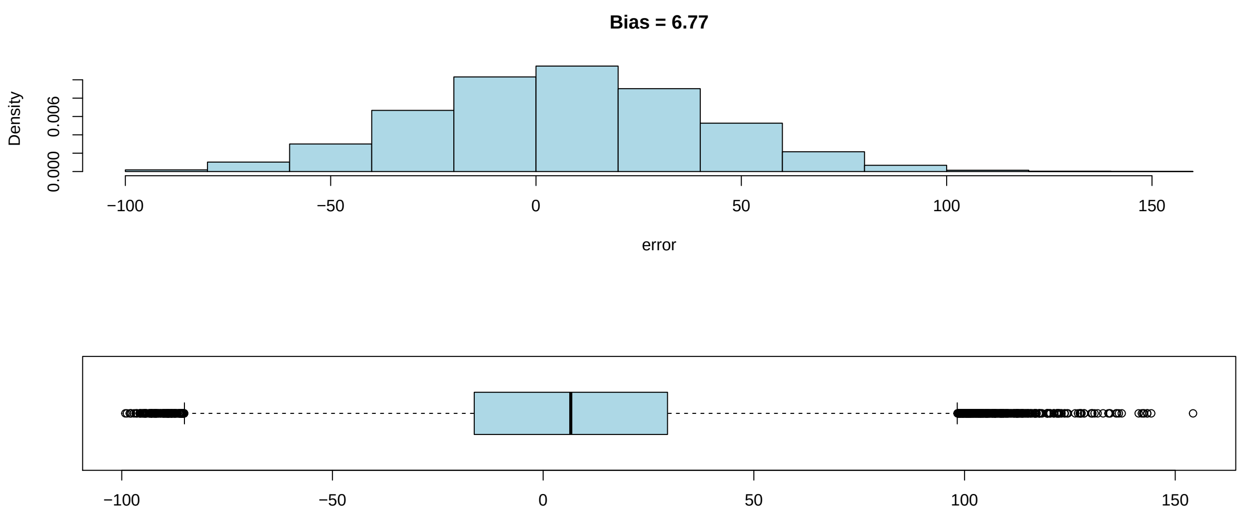


Figure 3: Histogram and boxplot of the observed errors for the case σ= 25

3.3 The case $\sigma = 35$

Proceeding as before for the case $\sigma = 35$ we get the following results depicted in Figure 3.

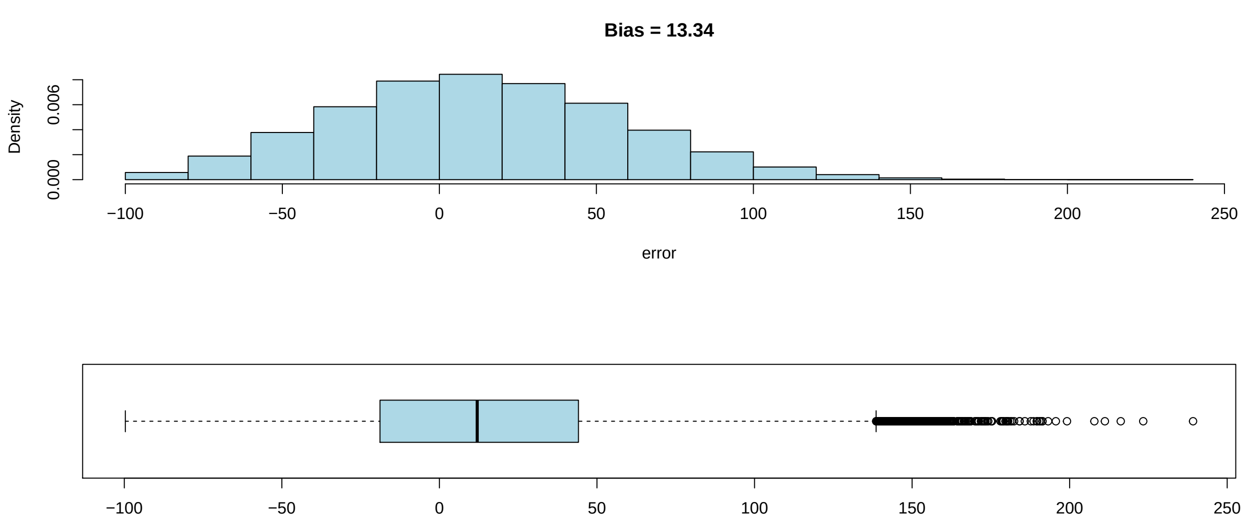


Figure 4: Histogram and boxplot of the observed errors for the case σ= 35

sigma	bias
15.00	2.27
25.00	6.77
35.00	13.34

Table 1: Quick overview sigma versus bias

4 Simulation results for uniformly distributed errors

4.1 The case $a = b = 25$

We proceed as in the previous section but this time assume that the errors are uniformly distributed on the interval $[-25, 25]$.

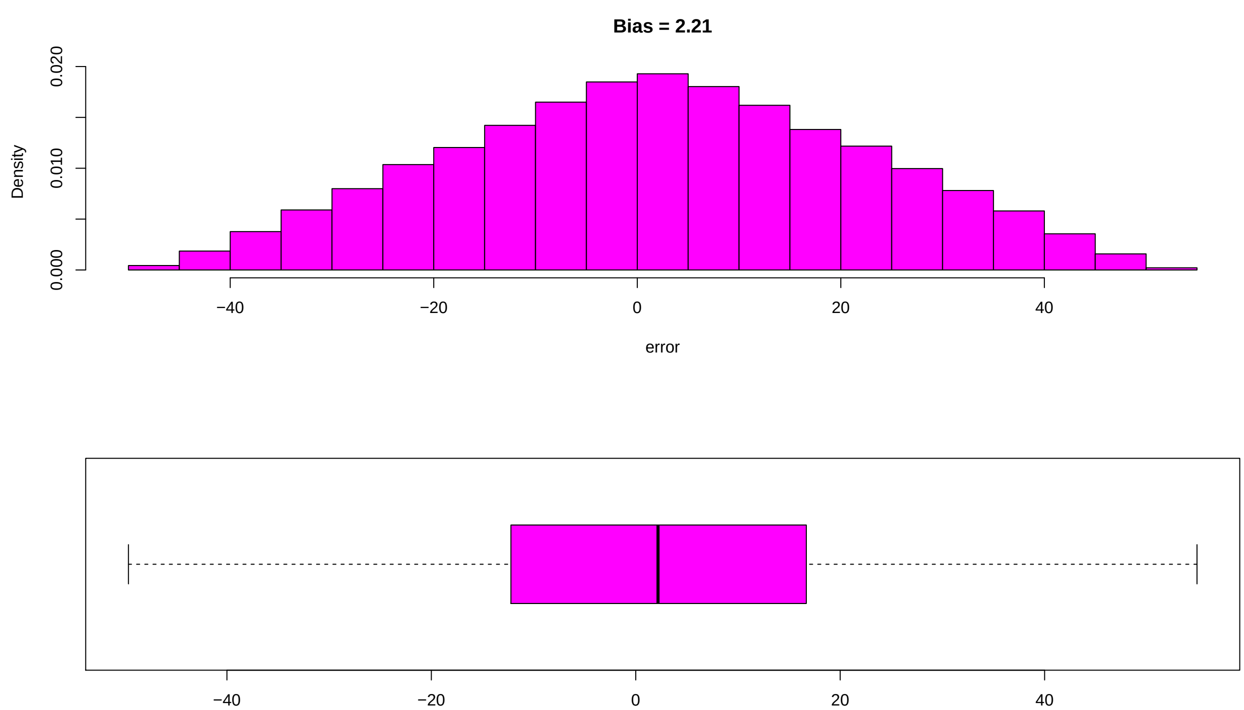
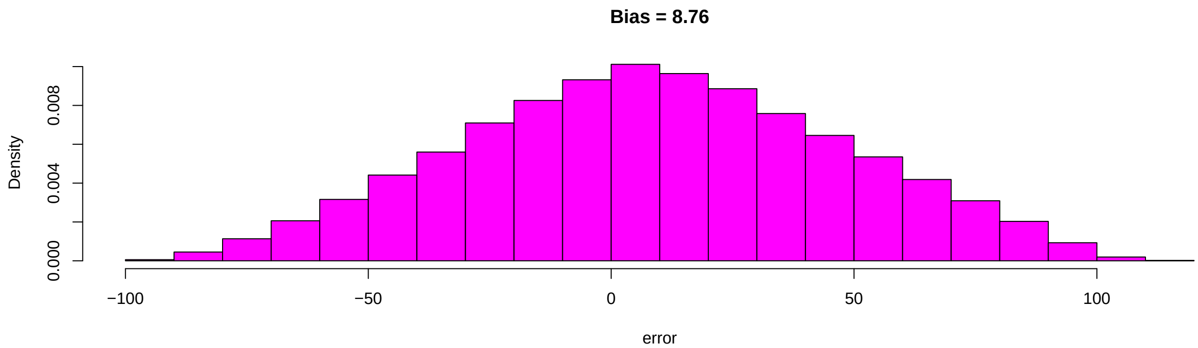


Figure 5: Histogram and boxplot of the observed errors for the case a= 25

4.2 The case $a = b = 50$

We proceed as in the previous section but this time assume that the errors are uniformly distributed on the interval $[-50, 50]$.



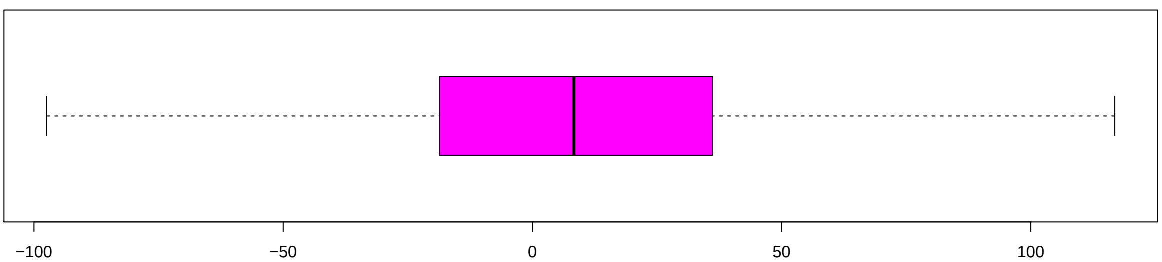


Figure 6: Histogram and boxplot of the observed errors for the case a= 50

4.3 The case $a = b = 75$

We proceed as in the previous section but this time assume that the errors are uniformly distributed on the interval $[-75, 75]$.

sigma	bias
25.00	2.27
50.00	2.21
75.00	13.34

Table 2: Quick overview sigma versus bias

A more systematic study of this GPS-bias problem can be found in the recently published paper [1].

References

[1] P. Ranacher, R. Brunauer, W. Trutschnig, S. Van der Spek, S. Reich: Why GPS makes distances bigger than they are, International Journal of Geographical Information Science 30, 316-333 (2016) (open access)

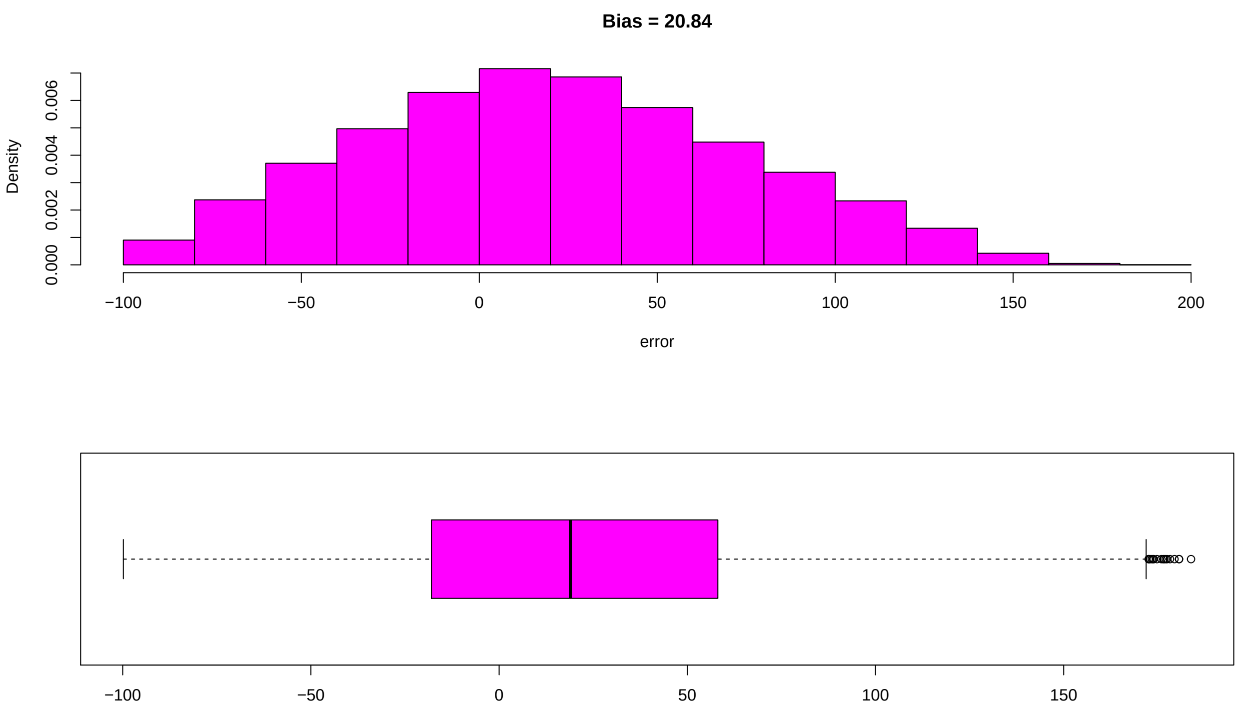


Figure 7: Histogram and boxplot of the observed errors for the case a= 75